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STRESSES AND BUCKLING IN THIN DOMES UNDER INTERNAL PRESSURE

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CONTENTS

STRESSES AND BUCKLING IN THIN DOMES UNDER INTERNAL PRESSURE

I. INTRODUCTION

The response of thin structural domes subjected to internal pressure (acting on the concave surface) has been a recent subject of concentrated study primarily because of the need for accurate design of such domes when used as pressure vessel end closures in missiles and other such structures. In these design situations the importance of the capability for accurate and reliable prediction of structural response is easily visualized.

Another possible application of such analysis in a situation unique to pressure. forming of domes is in the examination of the structural behavior of a dome-shaped vessel while it is in the process of being formed. Here the response of interest would be buckling instability stemming from circumferential compressive stresses produced during forming, since this type of abortive behavior would lead to an unsatisfactory finished product.

Dome buckling as a result of circumferential compression associated with "flange pull-in" is a well-documented fact and a current study problem. Although buckling without flange pull-in has not been observed, there appears to be no fundamental behavioral law associated with explosive forming that would definitely preclude this behavior. Therefore, similar buckling as the result of circumferential compressive stresses produced without flange pull-in should be considered a possibility unless shown to be physically impossible.

The latter, "during forming", analysis is directly related to explosive forming technology itself and requires continued intensive study. The present state of art in the analysis of this dynamic problem would limit treatment to the consideration of instantaneous physical states of the progressively forming dome and would require initial information inputs regarding instantaneous geometric shapes and material condition. These initial information inputs are themselves largely unknown and the subject of current intensive study.

The analysis and design of finished domes for structural application has less direct relationship to the technology of explosive forming. However, the goals of explosive forming studies in terms of control of geometry and material condition must be guided by considerations of the relative performance capabilities as well as the cost of explosively formed domes. These relative capabilities can be established only by comparison with the performance of domes produced by other competitive methods and by further comparison with theoretically achievable performance.

Recent comparisons of results of theoretical and experimental mechanics studies on the behavior of thin torispherical domes used for pressure vessels have shown that, for the particular case of static internal pressure loading, the stress and buckling response of theoretical domes, as predicted by recently developed nonlinear mathematical solutions, agrees almost perfectly with the actual behavior observed in carefully machined metallic domes. This means that the necessary basis for accurate design and efficient utilization

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of internally pressurized machined domes exists; a basis that can be exploited in pressure vessel design to obtain optimum weight machined domes for those applications where weight:strength ratios are an important factor.

Of practical significance to those in the explosive forming business is the relative behavior of domes manufactured by machining and those manufactured by forming methods, such as explosive forming, which are known to introduce significant variations in geometry and material properties. These variations, as they occur in spun domes, have been observed to produce important variations in overall structural behavior and unpredictability of structural strengths.

The extent to which explosively formed domes are found usable in applications where weight is an important consideration - and most aerospace domes fall in this category - must depend to no small degree on their quality and uniformity as compared to machined domes. To ascertain the relative merit of the products of these two competing methods of fabrication it would be necessary to perform test programs on explosively formed domes similar to the programs carried out on machined domes and described here.

The content of this presentation includes experimental data, observations, and analytical results gathered primarily during a comprehensive study of the behavior of thin domes of torispherical geometry carried out at U. S. Army Materials Research Agency. Some of this information is now available in the open literature. Other more recent findings are currently undergoing final processing preparatory to publication as in-house reports and ultimately in the open literature.

II. DOME RESPONSE TO INTERNAL PRESSURE

A. Stresses

Figure 1 shows schematically the specific thin torispherical dome considered in this treatment. In this application the dome is subjected to net internal pressure acting on the concave surface. The major dimensions shown define the geometry of this type of dome.

Under the action of the applied pressure, stresses in deep domes are primarily tensile. For example, for a thin hemispherically shaped dome a state of almost uniform biaxial tension would develop over most of the dome.

However, for shallower geometries the stresses deviate from uniform tension such that for certain shallow geometries meridional compressive stresses occur in some portions of the dome. For ellipsoidal shells, for example, according to Clark and Reissner (1957) circumferential compressive stresses will occur in geometries in which the ratio of minor axis to major axis is $1:\sqrt{2}$ or less. In torispherical domes of similar geometry compressive stresses can also occur. As the geometry becomes even shallower and depending on the thickness of the dome material, it is possible for compressive stresses
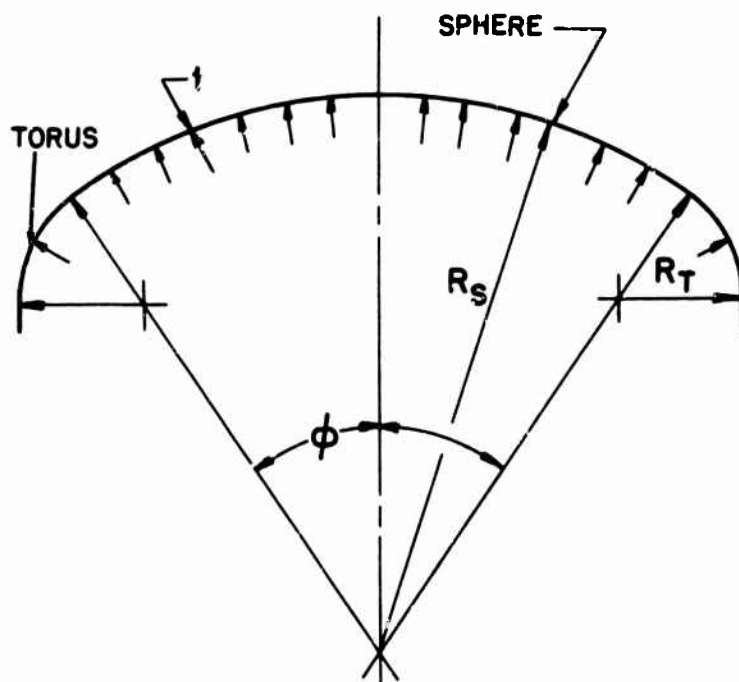
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Figure 1. TORISPHERICAL SHELL UNDER INTERNAL PRESSURE
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of sufficiently large magnitude to occur over a sufficiently large portion of
a dome to produce elastic buckling instability.

The overall stress distribution for the various geometries is compli-
cated by the contributions of bending stresses at geometry changes and by the
important influence of nonlinear behavior which will be discussed in greater
detail later.

B. Failure Modes

Failure of a dome under internal pressure can occur as elastic buckling
or material yielding with the latter leading to material failure or plastic
buckling.

Elastic buckling develops in the manner shown in Figure 2. The test
dome in this figure was vacuum molded of clear Vinylite plastic and was sub-
jected to static internal hydraulic pressure to produce the initial buckle
shown.

With increasing pressure additional buckles are produced as shown in
Figure 3 until the fairly uniformly distributed pattern of buckles shown in
Figure 4 is attained.

Material yielding will be the governing failure mode for larger thick-
nesses depending on the strength of the material. In addition, of course,
the yield failure mode will govern for those shell geometries in which
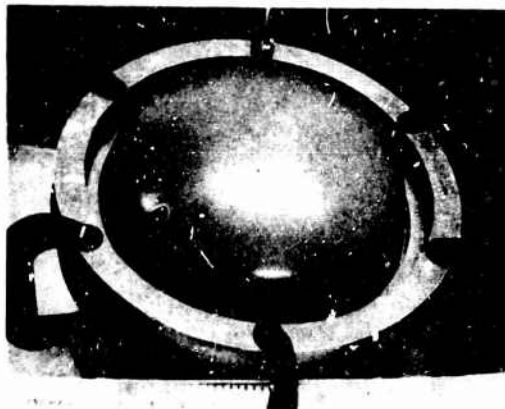compressive stresses do not occur.

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Figure 2. INITIAL BUCKLING OF TEST BULKHEAD
p = p critical

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Figure 3. ADDITIONAL BUCKLING OF TEST
BULKHEAD. p slightly greater than p critical

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Figure 4. ADDITIONAL BUCKLING OF TEST
BULKHEAD. p much greater than p critical

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III. FAILURE PREDICTION

A. Early Buckling Studies Using Vinylite Models

Early studies of the behavior of torispherical domes under internal pressure were motivated by the buckling of two 105-inch-diameter prototype domes used in fuel containers of a large Jupiter missile. Each of these domes was fabricated by welding togethe. a spun toroidal knuckle section and several formed sectors making up the spherical cap. The material utilized was 5086 H-34 aluminum alloy. Toroidal radius (R_T) was 18.2 inches and central angle (ϕ) was 35 degrees.

Initial investigation as reported by Adachi and Benicek (1964) utilized one-tenth scale Vinylite plastic domes, as shown in Figures 2, 3, and 4, to obtain empirical curves for predicting elastic buckling failure. These plastic models were fabricated using a vacuum forming procedure which produced accurate geometry and thickness with no detectable residual stress.

Figure 5 is a plot of the buckling behavior of these Vinylite domes showing the least-squares fit curves of an empirical power relationship and also the data points for the two prototype domes.

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For accurate prediction the two prototype data points should have fallen much closer to the upper line corresponding to $\phi = 35$ degrees and $R_T = 1.82$ inches. Because possible strain rate effects made the validity of scaling modulus uncertain and because of the lack of information on the stress distributions and magnitudes in these domes at the time of these tests, the exact reason for the discrepancy between predicted and actual prototype dome behavior could not be firmly determined. It was tentatively concluded that the prototypes failed at a pressure much lower than that predicted for elastic buckling because of material yielding which precipitated plastic buckling. (This conclusion was based on membrane analysis and bonded SR-4 strain gage measurements made on Vinylite bulkheads. Both were considered very approximate but did show stress magnitudes that supported the conclusion.)

The need for stress analysis to provide a basis for determining limiting pressures for yield failure was definitely indicated, and more accurate stress analysis and strain measurements were made the subject of further study. To determine the validity of modulus scaling in elastic buckling predictions, further testing of metallic domes was also undertaken.

A mathematical elastic buckling solution by Mescall (1962) utilizing linear small deflection theory was available at this time. Comparisons of this theory and the experimentally determined elastic buckling curves showed marked lack of quantitative agreement as shown in Figure 6. At this stage of the study it was uncertain whether the experimental results or the theoretical results (or both) were the cause of the disagreement.
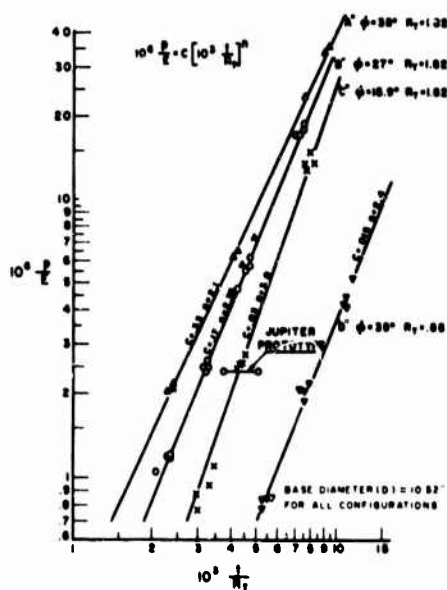

Figure 5. BUCKLING PRESSURE VERSUS THICKNESS FOR VARIOUS BULKHEAD CONFIGURATIONS
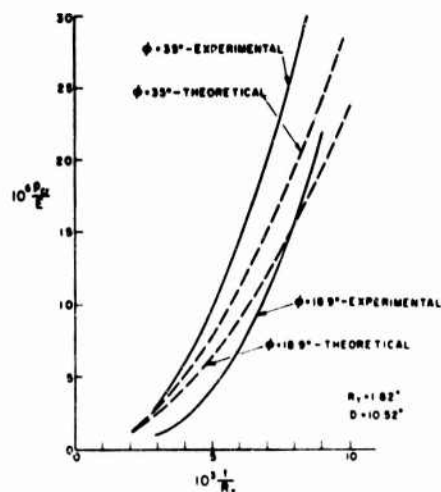
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Figure 6. TORISPHERICAL SHELL BUCKLING - EXPERIMENTAL RESULTS (Adachi & Benicek, 1964) AND THEORY (Mescall, 1962)

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B. Metallic Dome Tests

1. Spun Metal Domes

Continued experimental studies first utilized metallic domes produced by spinning methods. These domes were twice the diameter (21.0 inches) of the earlier plastic domes, had thicknesses of approximately 0.015 inch, and were fabricated from aluminum sheet of 6061 alloy. Tests were performed as shown in Figure 7.

Accurate stress measurement utilizing bonded SR-4 strain gages produced data as shown in Figure 8. This figure compares experiment with linear theory and reveals the significant differences in the important high stressed regions that led to continued uncertainty in both the analytical and experimental results. A feature of the experimental procedure that was considered of major importance and concern was the variability in the overall quality of the models that were produced by spinning. Variations in thickness, geometry, material condition, and residual stress condition were known to exist as a natural consequence of the spinning process although the exact measurement of these variations (other than thickness) was not attempted. "As-spun" domes had no visually apparent deviation. However, attempts to stress-relieve some models resulted in small but readily visible geometric variations.

These variations were expected to be of even greater significance in the buckling behavior of spun domes. As expected, buckling occurred at widely varying pressures far below the values predicted by the Vinylite prediction curves as shown later.

Figure 7. TEST SETUP FOR STRAIN MEASUREMENTS ON TORISPHERICAL SHELLS UNDER INTERNAL PRESSURE LOADING
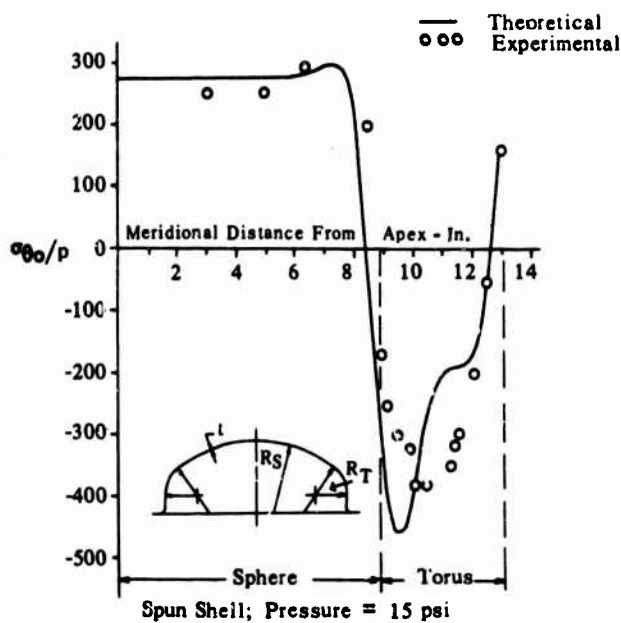
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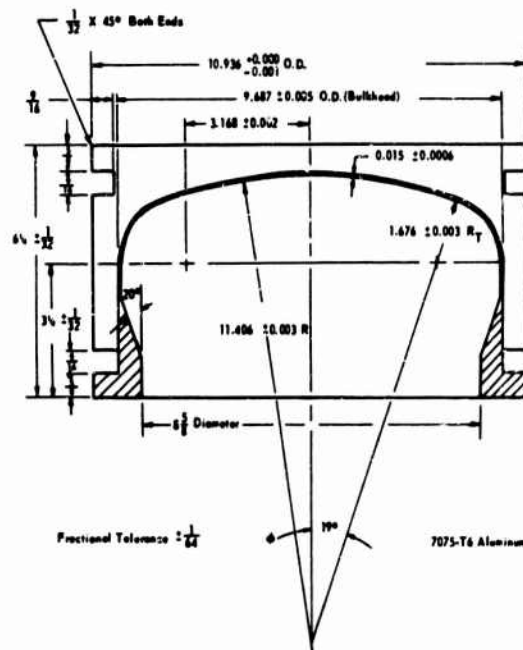Figure 8. COMPARISON OF SPUN SHELL STRESSES WITH THEORETICAL

Figure 9. MACHINING DRAWING FOR MACHINED TORISPHERICAL SHELL

2. Machined Metal Domes

The continued uncertainty in the accuracy of the experimental results obtained led to the utilization of very carefully and accurately machined test domes. These domes were machined from solid heat-treated material as shown in Figure 9.

Machining was performed in several stages. As the thickness was reduced, low melting point (117 F) alloy was utilized to provide support for the skin against the pressure of the cutting tool. The overall procedure was successful in producing test domes with accurately controlled thickness and geometry of known material condition and free of serious residual stresses.

Several test domes were machined by this procedure utilizing Ti-6Al-6V-2Sn alloy at 160,000 psi yield strength for 5.18-inch-diameter domes and 7075-T6 aluminum alloy at 73,000 psi yield strength for 5.13-inch- and 9.68-inch-diameter domes.

Tests were performed to determine elastic buckling pressures and stress magnitudes and distribution.

3. Buckling Test Results for Metallic Domes

The results of buckling tests performed on spun and machined metallic domes are shown in Figure 10. On this figure are reproduced empirical prediction curves obtained from the earlier Vinylite model tests.

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The solid circles (●) are data points for machined metal domes corresponding to initial buckle formation. The smaller solid points correspond to development of second, third, and later buckle formation. The close agreement between the metal dome buckling and the empirical prediction curves shows that these curves accurately predict the behavior of carefully fabricated metallic domes. The data points shown by (⊗) marks correspond to the initial buckling of spun aluminum domes with geometries equivalent to the middle buckling curve (ϕ = 27 degrees).

In contrast to the agreement obtained with machined domes, the disagreement between predictive curve and spun dome behavior is very great and can be attributed only to the lower quality of fabrication resulting from the spinning procedure.

Figure 10. COMPARISON OF METALLIC SHELL BUCKLING DATA WITH PLASTIC SHELL BUCKLING CURVES

4. Stress Measurements on Machined Domes

Superimposed on the buckling curves of Figure 10 are approximate limiting yield curves based on maximum shear theory for material yield for 160,000 psi yield strength Ti-6Al-6V-2Sn alloy and 73,000 psi yield strength 7075-T6 aluminum for ϕ = 18.9 degrees. These yield curves are based on linear stress analysis results and together with the elastic buckling curve should form limiting boundaries on the pressure-carrying capability of these metallic domes. The uppermost data point for the aluminum 5.18-inch-diameter dome is well outside the limiting boundaries and is an anomaly which makes the validity of these yield curves suspect.

Carefully planned strain measurements were made on an accurately machined aluminum test dome 0.015 inch thick to provide indisputably accurate data on stress distribution and magnitudes for use as the basis for evaluating the validity of analytical methods.

Figure 11 shows a mock-up utilized to plan the location of electrical resistance strain gages. The pattern shown was reproduced (in back-to-back fashion) on the inside surface of the test dome. Supplementary tests were performed to ascertain the effect of gage installation on the accuracy of strain readings. It was found that gage error in measuring bending strains on metal 0.007 inch thick was on the order of 10 percent. The 0.015-inch thickness of the test dome was adopted to reduce this error. (The actual final net effect on stress values was only a few percent since bending stresses were only a fraction of the total stress). Figure 12 shows the test dome and base fixture prepared for assembly and test.
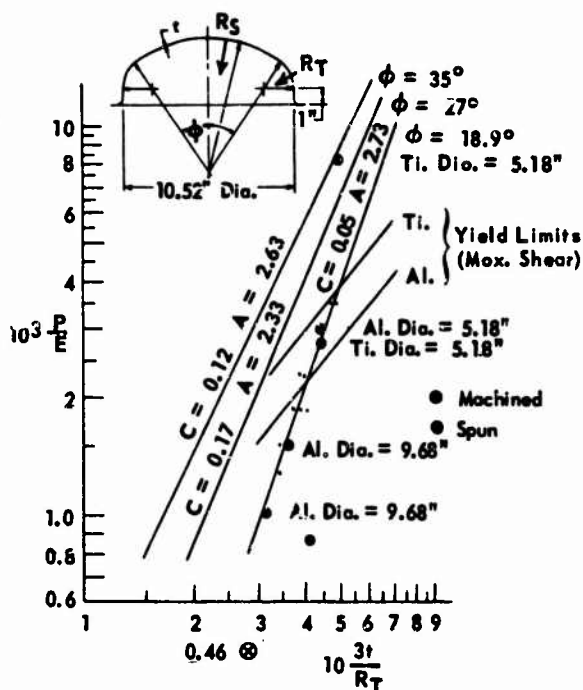
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Examination of the test results revealed very interesting and enlightening information. The major portion of the spherical cap showed a linear relationship between stress and applied internal pressure. However, the data from the toroidal portions and the portions of the spherical cap immediately adjacent to the toroidal portion showed definite nonlinear behavior as exemplified by the plot of Figure 13.

Meridional distributions of stresses are shown in Figures 14 and 15. In these figures the data represents the values of σ/E for only the specified pressure value since the nonlinear stress-pressure relationship does not permit a single value for the σ/E ratio.

5. Comparison of Experimental and Theoretical Stresses

The results of analytical linear analysis are also plotted and can be seen to deviate drastically from the measured results. For lower pressure levels the relative magnitude of the deviations decreases. For higher pressure levels the relative deviations increase as the result of the nonlinearity.

Discussions with Thurston (1966) regarding results of nonlinear mathematical analyses on similar geometries indicated that his analyses had shown similar nonlinear stress-pressure relationships. Results of the mathematical procedure of Thurston (Thurston and Holston 1966) and similar analyses of Mescall (1966) applied to the specific dimensions of the experimental dome provided the almost perfect agreement shown in Figures 14 and 15.

The large deviation between the linear analyses and the actual nonlinear behavior indicates that the yield limit curves based on linear theory

Figure 11. MOCKUP OF STRAIN GAGE LOCATIONS FOR MACHINED MODEL

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Figure 12. MACHINED 7075-T6 Al TORISPHERICAL SHELL MODEL - 9.68" BASE DIAMETER - WITH BASE FIXTURE

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(Figure 10) are too low. Correction of these limiting curves (not accomplished as yet) should also correct the anomalous condition noted earlier regarding the location of the uppermost aluminum dome data point relative to the yield curve.

Of greater significance is the closeness of the agreement between the nonlinear analysis and the behavior of carefully machined domes. This indicates very accurate prediction and control of stress conditions in machined domes is possible, thus providing the designer with a powerful tool for optimizing designs in machined domes.

6. Nonlinear Buckling Analyses

The extension of nonlinear analysis to the elastic buckling of torispherical domes and ellipsoidal domes has been carried out by Thurston and Holston (1966). Comparisons of theoretical torispherical results with experimental buckling curves reveal very good agreement as shown by Figure 16.

Additional experimental data covering a broader range of torispherical geometries are available. Currently the Thurston analysis is being applied to these additional geometries. Also, experimental buckling studies of ellipsoidal shells have been initiated at Army Materials Research Agency.

IV. CONCLUDING REMARKS

The foregoing examination into the state of art in analysis of the behavior of constant thickness torispherical domes under internally applied pressure shows that extremely accurate prediction of stresses and critical buckling pressures can be accomplished by current mathematical techniques. In addition, fabrication by normal machining procedures has been shown to be of sufficiently high quality to ensure reproducible, close-to-theoretical, structural behavior.

However, the variable quality of fabrication inherent in the spinning process leads to erratic behavior and renders accurate, efficient design impractical. This shortcoming reduces the usefulness of domes formed by this process in those applications where efficient use of structural weight is an important factor.

Designers will naturally tend to make similar comparisons when considering the use of domes fabricated by other forming methods including explosive forming. In weight-critical applications, comparative structural performance may well be of primary importance with comparative cost of lesser importance.

Although the comparisons made herein apply strictly to constant thickness torispherical domes, the capability of machining procedure to provide variable thickness (constant strength) domes will certainly be an added advantage once the state of art in analysis of variable-thickness domes attains the usable level. In addition, there is every reason to believe that other forms
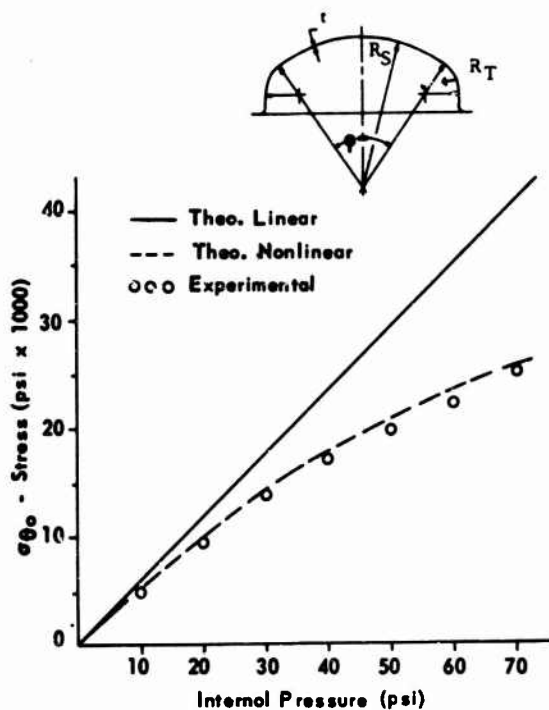
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Figure 13. PLOT OF TYPICAL EXPERIMENTAL STRESSES AND THEORETICAL LINEAR AND NONLINEAR RESULTS
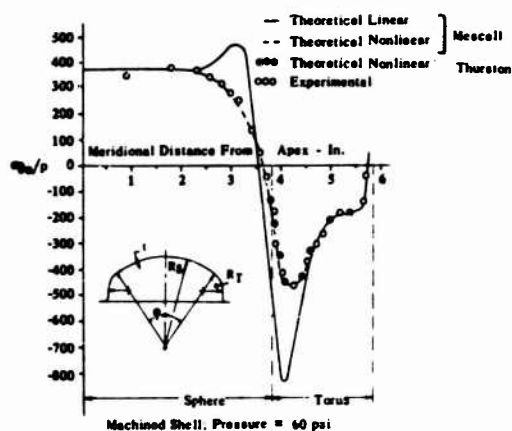

Figure 14. COMPARISON OF EXPERIMENTAL AND THEORETICAL STRESSES - EXTERIOR CIRCUMFERENTIAL STRESSES ALONG A MERIDIAN
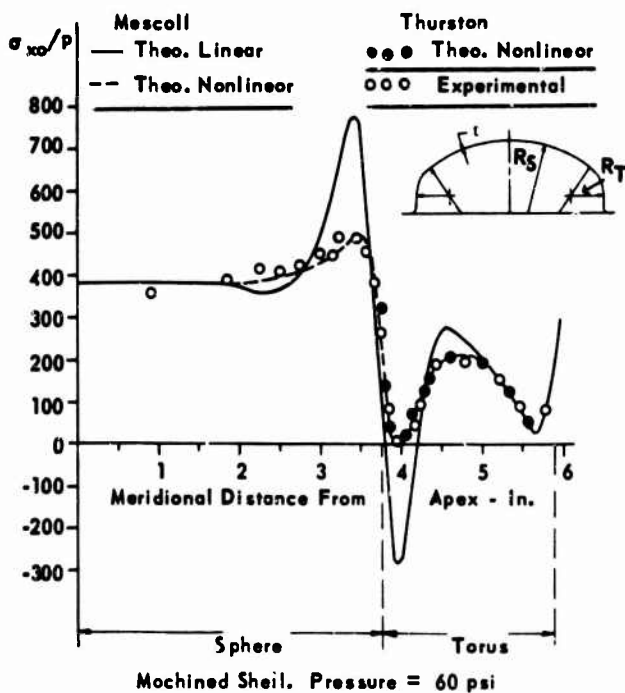

Figure 15. COMPARISON OF EXPERIMENTAL AND THEORETICAL STRESSES - EXTERIOR MERIDIONAL STRESSES ALONG A MERIDIAN
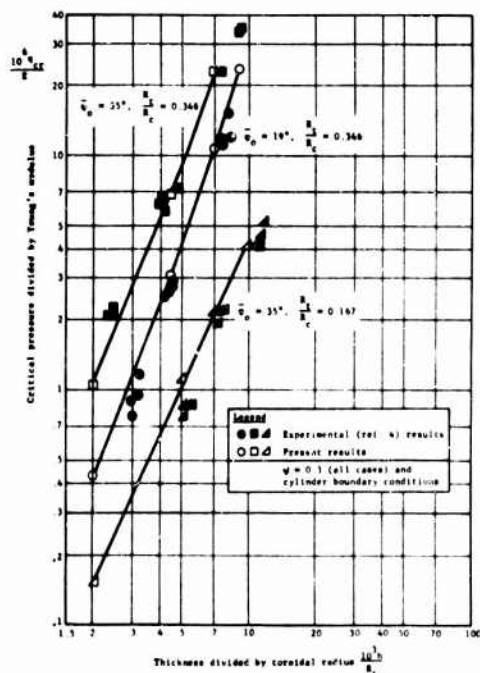

Figure 16. COMPARISON OF THEORETICAL NONLINEAR BUCKLING (Thurston and Holston, 1966) WITH EXPERIMENTAL BUCKLING RESULTS (Adachi and Benicek, 1964) (Reproduced from Thurston and Holston, 1966)

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of domes will show similar predictable behavior when experiment and theory are compared.

The importance of developing the techniques of explosive forming to the degree which permits accurate control and close reproducibility cannot therefore be over-emphasized. It appears that those·whose interest lies in this technique should consider directing some research effort toward evaluating overall performance of their product in order to define the desirable goals for dome geometry, thickness, and fabrication tolerances.

BIBLIOGRAPHY

ADACHI, J., and BENICEK, M., (1964). *Buckling of Torispherical Shells under Internal Pressure,* Experimental Mechanics, August 1964.

CLARKE, R. A., and REISSNER, E., (1957). *On Stresses and Deformations of Ellipsoidal Shells Subject to Internal Pressure,* Journal of Mechanics and Physics of Solids, v. 6, 1957, p. 63-70.

MESCALL, J., (1962). *Stability of Thin Torispherical Shells under Uniform Pressure,* "Collected Papers on Instability of Shell Structures", National Aeronautics and Space Administration, NASA TN D-1510, December 1962, p. 671-692.

MESCALL, J., (1966). *Numerical Solution of the Nonlinear Axisymmetric Equations for Shells of Revolution,* U. S. Army Materials Research Agency, AMRA TR 66-11, May 1966.

THURSTON, G. A., (1966). Informal discussions, G. Thurston, A. Holston, and J. Adachi, August 1966.

THURSTON, G. A., and HOLSTON, A. A., JR., (1966). *Buckling of Cylindrical Shell End Closures by Internal Pressure,* National Aeronautics and Space Administration, NASA CR-540, July 1966.

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13. ABSTRACT

Review of the state-of-the-art in analysis of the behavior of thin domes under internal pressures, specifically constant thickness torispherical domes, shows that extremely accurate prediction of stresses and critical buckling pressures for carefully machined domes can be achieved.

However, forming procedures such as spinning and explosive forming produce domes of such variable quality and erratic, unpredictable behavior that accurate, efficient design is not possible.

Thus an important area of study for explosive forming advocates should be development of techniques which permit accurate control and close reproducibility.
                                                                      (Author)

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